4.4 Solving Two-Step Inequalities

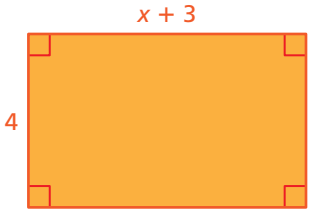
Essential Question How can you use an inequality to describe the dimensions of a figure?

1 ACTIVITY: Areas and Perimeters of Figures

Work with a partner.

- **Use the given condition to choose the inequality that you can use to find the possible values of the variable. Justify your answer.**
- **Write four values of the variable that satisfy the inequality you chose.**

a. You want to find the values of x so that the area of the rectangle is more than 22 square units.



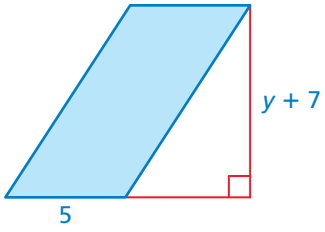
$4x + 12 > 22$ $\quad$ $4x + 3 > 22$

$4x + 12 \geq 22$ $\quad$ $2x + 14 > 22$

b. You want to find the values of x so that the perimeter of the rectangle is greater than or equal to 28 units.

$x + 7 \geq 28$ $\quad$ $4x + 12 \geq 28$ $\quad$ $2x + 14 \geq 28$ $\quad$ $2x + 14 \leq 28$

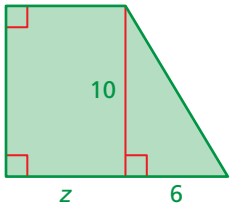
c. You want to find the values of y so that the area of the parallelogram is fewer than 41 square units.



$5y + 7 < 41$ $\quad$ $5y + 35 < 41$

$5y + 7 \leq 41$ $\quad$ $5y + 35 \leq 41$

d. You want to find the values of z so that the area of the trapezoid is at most 100 square units.



$5z + 30 \leq 100$ $\quad$ $10z + 30 \leq 100$

$5z + 30 < 100$ $\quad$ $10z + 30 < 100$

Inequalities

In this lesson, you will

- solve multi-step inequalities.
- solve real-life problems.

2 ACTIVITY: Volumes of Rectangular Prisms

Work with a partner.

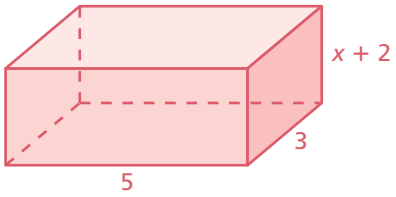
- **Use the given condition to choose the inequality that you can use to find the possible values of the variable. Justify your answer.**
- **Write four values of the variable that satisfy the inequality you chose.**

Math Practice

State the Meaning of Symbols

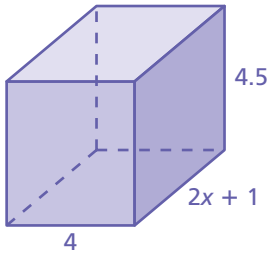
What inequality symbols do the phrases *at least* and *no more than* represent? Explain.

a. You want to find the values of x so that the volume of the rectangular prism is at least 50 cubic units.



$15x + 30 > 50$ | $x + 10 \geq 50$ | $15x + 30 \geq 50$ | $15x + 2 \geq 50$

b. You want to find the values of x so that the volume of the rectangular prism is no more than 36 cubic units.



$8x + 4 < 36$ | $36x + 18 < 36$ | $2x + 9.5 \leq 36$ | $36x + 18 \leq 36$

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you use an inequality to describe the dimensions of a figure?

4. Use what you know about solving equations and inequalities to describe how you can solve a two-step inequality. Give an example to support your explanation.

Practice Use what you learned about solving two-step inequalities to complete Exercises 3 and 4 on page 150.

4.4 Lesson

Check It Out
Lesson Tutorials
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You can solve two-step inequalities in the same way you solve two-step equations.

EXAMPLE 1 Solving Two-Step Inequalities

a. Solve $5x - 4 \geq 11$. Graph the solution.

$5x - 4 \geq 11$ — Write the inequality.

Step 1: Undo the subtraction.

$+4 \quad +4$ — Addition Property of Inequality

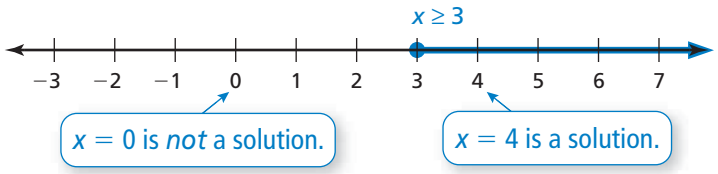
$5x \geq 15$ — Simplify.

Step 2: Undo the multiplication.

$\frac{5x}{5} \geq \frac{15}{5}$ — Division Property of Inequality

$x \geq 3$ — Simplify.

∴ The solution is $x \geq 3$.



b. Solve $\frac{b}{-3} + 4 < 13$. Graph the solution.

$\frac{b}{-3} + 4 < 13$ — Write the inequality.

Step 1: Undo the addition.

$-4 \quad -4$ — Subtraction Property of Inequality

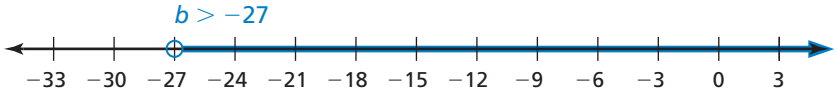
$\frac{b}{-3} < 9$ — Simplify.

Step 2: Undo the division.

$-3 \cdot \frac{b}{-3} > -3 \cdot 9$ — Use the Multiplication Property of Inequality. Reverse the inequality symbol.

$b > -27$ — Simplify.

∴ The solution is $b > -27$.



On Your Own

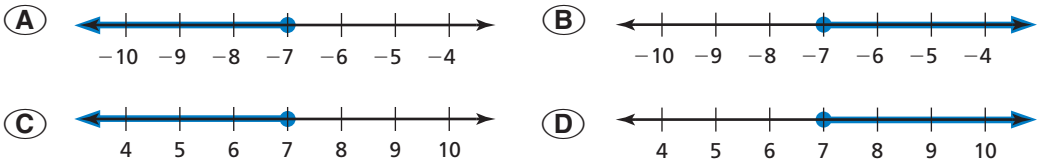
Now You're Ready
Exercises 5–10

Solve the inequality. Graph the solution.

1. $6y - 7 > 5$ **2.** $4 - 3d \geq 19$ **3.** $\frac{w}{-4} + 8 > 9$

EXAMPLE 2 Graphing an Inequality

Which graph represents the solution of $-7(x + 3) \leq 28$?



$-7(x + 3) \leq 28$ — Write the inequality.

$-7x - 21 \leq 28$ — Distributive Property

Step 1: Undo the subtraction. → $+21 \quad +21$ — Addition Property of Inequality

$-7x \leq 49$ — Simplify.

Step 2: Undo the multiplication. → $\frac{-7x}{-7} \geq \frac{49}{-7}$ — Use the Division Property of Inequality. Reverse the inequality symbol.

$x \geq -7$ — Simplify.

∴ The correct answer is (B).

EXAMPLE 3 Real-Life Application

Progress Report	
Month	**Pounds Lost**
1	12
2	9
3	5
4	8

A contestant in a weight-loss competition wants to lose an average of at least 8 pounds per month during a 5-month period. How many pounds must the contestant lose in the fifth month to meet the goal?

Write and solve an inequality. Let x be the number of pounds lost in the fifth month.

$\frac{12 + 9 + 5 + 8 + x}{5} \geq 8$ — The phrase *at least* means *greater than or equal to.*

$\frac{34 + x}{5} \geq 8$ — Simplify.

$5 \cdot \frac{34 + x}{5} \geq 5 \cdot 8$ — Multiplication Property of Inequality

$34 + x \geq 40$ — Simplify.

$x \geq 6$ — Subtract 34 from each side.

∴ So, the contestant must lose at least 6 pounds to meet the goal.

Remember

In Example 3, the average is equal to the sum of the pounds lost divided by the number of months.

On Your Own

Now You're Ready
Exercises 12–17

Solve the inequality. Graph the solution.

4. $2(k - 5) < 6$ **5.** $-4(n - 10) < 32$ **6.** $-3 \leq 0.5(8 + y)$

7. WHAT IF? In Example 3, the contestant wants to lose an average of at least 9 pounds per month. How many pounds must the contestant lose in the fifth month to meet the goal?

4.4 Exercises

Check It Out
Help with Homework
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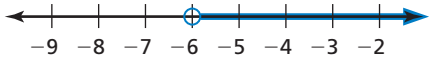
Vocabulary and Concept Check

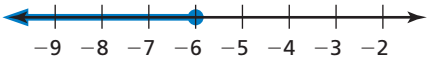
1. **WRITING** Compare and contrast solving two-step inequalities and solving two-step equations.
2. **OPEN-ENDED** Describe how to solve the inequality $3(a + 5) < 9$.

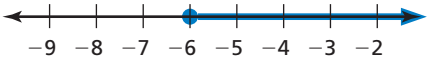
Practice and Problem Solving

Match the inequality with its graph.

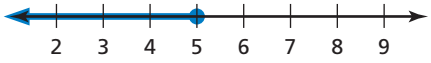
3. $\frac{t}{3} - 1 \geq -3$

A. 

B. 

C. 

4. $5x + 7 \leq 32$

A. 

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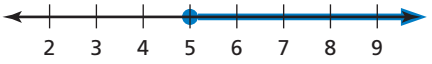
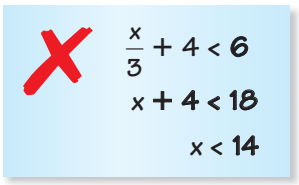
C. 

Solve the inequality. Graph the solution.

1 5. $8y - 5 < 3$ 6. $3p + 2 \geq -10$ 7. $2 > 8 - \frac{4}{3}h$

8. $-2 > \frac{m}{6} - 7$ 9. $-1.2b - 5.3 \geq 1.9$ 10. $-1.3 \geq 2.9 - 0.6r$

11. **ERROR ANALYSIS** Describe and correct the error in solving the inequality.



Solve the inequality. Graph the solution.

2 12. $5(g + 4) > 15$ 13. $4(w - 6) \leq -12$ 14. $-8 \leq \frac{2}{5}(k - 2)$

15. $-\frac{1}{4}(d + 1) < 2$ 16. $7.2 > 0.9(n + 8.6)$ 17. $20 \geq -3.2(c - 4.3)$

18. **UNICYCLE** The first jump in a unicycle high-jump contest is shown. The bar is raised 2 centimeters after each jump. Solve the inequality $2n + 10 \geq 26$ to find the number of additional jumps needed to meet or exceed the goal of clearing a height of 26 centimeters.

Solve the inequality. Graph the solution.

19. $9x - 4x + 4 \geq 36 - 12$

20. $3d - 7d + 2.8 < 5.8 - 27$

21. **SCUBA DIVER** A scuba diver is at an elevation of -38 feet. The diver starts moving at a rate of -12 feet per minute. Write and solve an inequality that represents how long it will take the diver to reach an elevation deeper than -200 feet.

22. **KILLER WHALES** A killer whale has eaten 75 pounds of fish today. It needs to eat at least 140 pounds of fish each day.

a. A bucket holds 15 pounds of fish. Write and solve an inequality that represents how many more buckets of fish the whale needs to eat.

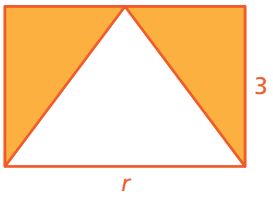
b. Should the whale eat *four* or *five* more buckets of fish? Explain.

23. **REASONING** A student theater charges \$9.50 per ticket.

a. The theater has already sold 70 tickets. Write and solve an inequality that represents how many more tickets the theater needs to sell to earn at least \$1000.

b. The theater increases the ticket price by \$1. Without solving an inequality, describe how this affects the total number of tickets needed to earn at least \$1000.

24. Problem Solving For what values of r will the area of the shaded region be greater than or equal to 12 square units?



Fair Game Review What you learned in previous grades & lessons

Find the missing values in the ratio table. Then write the equivalent ratios. *(Skills Review Handbook)*

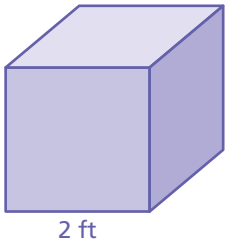
25.

Flutes	7		28
Clarinets	4	12	

26.

Boys	6	3	
Girls	10		50

27. **MULTIPLE CHOICE** What is the volume of the cube? *(Skills Review Handbook)*



Ⓐ $8\ ft^3$ Ⓑ $16\ ft^3$

Ⓒ $24\ ft^3$ Ⓓ $32\ ft^3$