4.3 Solving Inequalities Using Multiplication or Division

Essential Question How can you use multiplication or division to solve an inequality?

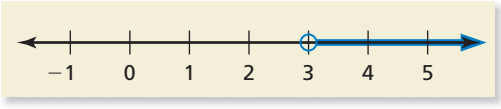
1 ACTIVITY: Using a Table to Solve an Inequality

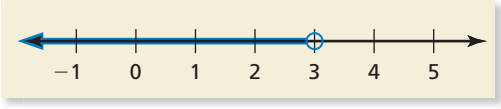
Work with a partner.

- **Copy and complete the table.**
- **Decide which graph represents the solution of the inequality.**
- **Write the solution of the inequality.**

a. $4x > 12$

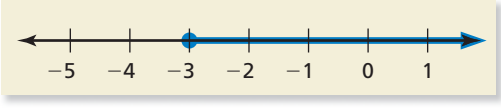
x	-1	0	1	2	3	4	5
$4x$							
$4x \stackrel{?}{>} 12$							

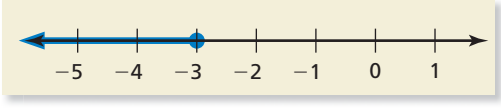




b. $-3x \leq 9$

x	-5	-4	-3	-2	-1	0	1
$-3x$							
$-3x \stackrel{?}{\leq} 9$							





Inequalities

In this lesson, you will

- solve inequalities using multiplication or division.
- solve real-life problems.

2 ACTIVITY: Solving an Inequality

Work with a partner.

a. Solve $-3x \leq 9$ by adding $3x$ to each side of the inequality first. Then solve the resulting inequality.

b. Compare the solution in part (a) with the solution in Activity 1(b).

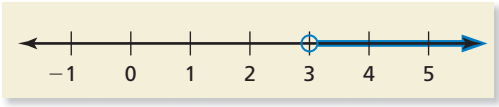
3 ACTIVITY: Using a Table to Solve an Inequality

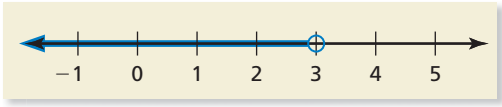
Work with a partner.

- **Copy and complete the table.**
- **Decide which graph represents the solution of the inequality.**
- **Write the solution of the inequality.**

a. $\frac{x}{3} < 1$

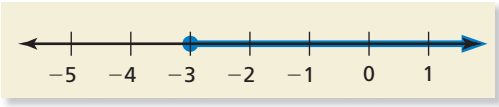
x	-1	0	1	2	3	4	5
$\frac{x}{3}$							
$\frac{x}{3} \stackrel{?}{<} 1$							

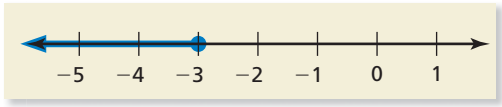




b. $\frac{x}{-4} \geq \frac{3}{4}$

x	-5	-4	-3	-2	-1	0	1
$\frac{x}{-4}$							
$\frac{x}{-4} \stackrel{?}{\geq} \frac{3}{4}$							





4 ACTIVITY: Writing Rules

Work with a partner. Use a table to solve each inequality.

a. $-2x \leq 10$ **b.** $-6x > 0$ **c.** $\frac{x}{-4} < 1$ **d.** $\frac{x}{-8} \geq \frac{1}{8}$

Write a set of rules that describes how to solve inequalities like those in Activities 1 and 3. Then use your rules to solve each of the four inequalities above.

Math Practice

Analyze Conjectures

When you apply your rules to parts (a)–(d), do you get the same solutions? Explain.

What Is Your Answer?

5. IN YOUR OWN WORDS How can you use multiplication or division to solve an inequality?

Practice Use what you learned about solving inequalities using multiplication or division to complete Exercises 4–9 on page 143.

4.3 Lesson

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Key Idea

Remember
Multiplication and division are inverse operations.

Multiplication and Division Properties of Inequality (Case 1)

Words When you multiply or divide each side of an inequality by the same *positive* number, the inequality remains true.

Numbers

$-4 < 6$ $\qquad$ $4 > -6$

$2 \cdot (-4) < 2 \cdot 6$ $\qquad$ $\frac{4}{2} > \frac{-6}{2}$

$-8 < 12$ $\qquad$ $2 > -3$

Algebra If $a < b$ and c is positive, then

$a \cdot c < b \cdot c$ and $\frac{a}{c} < \frac{b}{c}$.

If $a > b$ and c is positive, then

$a \cdot c > b \cdot c$ and $\frac{a}{c} > \frac{b}{c}$.

These properties are also true for $\leq$ and $\geq$.

EXAMPLE 1 Solving an Inequality Using Multiplication

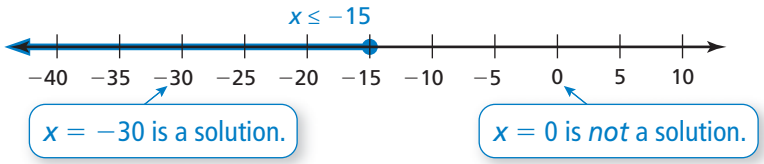
Solve $\frac{x}{5} \leq -3$. Graph the solution.

$\frac{x}{5} \leq -3$ Write the inequality.

Undo the division. → $5 \cdot \frac{x}{5} \leq 5 \cdot (-3)$ Multiplication Property of Inequality

$x \leq -15$ Simplify.

∴ The solution is $x \leq -15$.



On Your Own

Solve the inequality. Graph the solution.

1. $n \div 3 < 1$
2. $-0.5 \leq \frac{m}{10}$
3. $-3 > \frac{2}{3}p$

EXAMPLE 2 Solving an Inequality Using Division

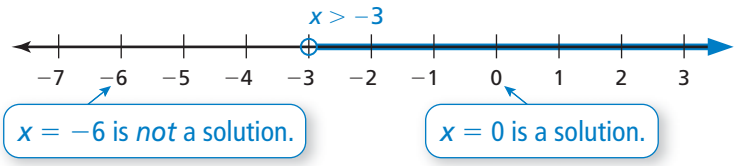
Solve $6x > -18$. Graph the solution.

$6x > -18$ Write the inequality.

Undo the multiplication. → $\frac{6x}{6} > \frac{-18}{6}$ Division Property of Inequality

$x > -3$ Simplify.

∴ The solution is $x > -3$.



On Your Own

Now You're Ready
Exercises 10–18

Solve the inequality. Graph the solution.

4. $4b \geq 2$ **5.** $12k \leq -24$ **6.** $-15 < 2.5q$

Key Idea

Multiplication and Division Properties of Inequality (Case 2)

Words When you multiply or divide each side of an inequality by the same *negative* number, the direction of the inequality symbol must be reversed for the inequality to remain true.

Numbers

$-4 < 6$ $\qquad$ $4 > -6$

$-2 \cdot (-4) > -2 \cdot 6$ $\qquad$ $\frac{4}{-2} < \frac{-6}{-2}$

$8 > -12$ $\qquad$ $-2 < 3$

Algebra If $a < b$ and c is negative, then

$a \cdot c > b \cdot c$ and $\frac{a}{c} > \frac{b}{c}$.

If $a > b$ and c is negative, then

$a \cdot c < b \cdot c$ and $\frac{a}{c} < \frac{b}{c}$.

These properties are also true for $\leq$ and $\geq$.

Common Error

A negative sign in an inequality does not necessarily mean you must reverse the inequality symbol.

Only reverse the inequality symbol when you multiply or divide both sides by a negative number.

EXAMPLE 3 Solving an Inequality Using Multiplication

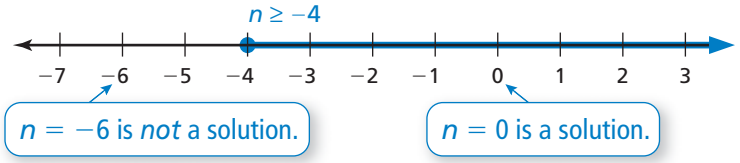
Solve $-\frac{3}{2}n \le 6$. Graph the solution.

$$-\frac{3}{2}n \le 6$$ Write the inequality.

$$-\frac{2}{3} \cdot \left(-\frac{3}{2}n\right) \ge -\frac{2}{3} \cdot 6$$ Use the Multiplication Property of Inequality. Reverse the inequality symbol.

$$n \ge -4$$ Simplify.

∴ The solution is $n \ge -4$.



On Your Own

Solve the inequality. Graph the solution.

7. $\frac{x}{-3} > -4$

8. $0.5 \le -\frac{y}{2}$

9. $-12 \ge \frac{6}{5}m$

10. $-\frac{2}{5}h \le -8$

EXAMPLE 4 Solving an Inequality Using Division

Solve $-3z > -4.5$. Graph the solution.

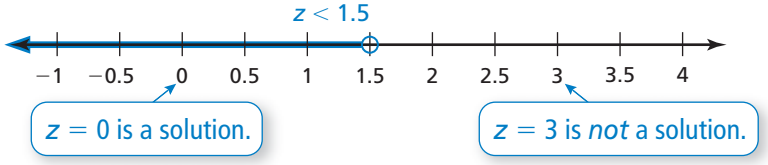
$$-3z > -4.5$$ Write the inequality.

Undo the multiplication. →

$$\frac{-3z}{-3} < \frac{-4.5}{-3}$$ Use the Division Property of Inequality. Reverse the inequality symbol.

$$z < 1.5$$ Simplify.

∴ The solution is $z < 1.5$.



On Your Own

Now You're Ready
Exercises 27–35

Solve the inequality. Graph the solution.

11. $-5z < 35$

12. $-2a > -9$

13. $-1.5 < 3n$

14. $-4.2 \ge -0.7w$

4.3 Exercises

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Vocabulary and Concept Check

1. **WRITING** Explain how to solve $\frac{x}{3} < -2$.
2. **PRECISION** Explain how solving $4x < -16$ is different from solving $-4x < 16$.
3. **OPEN-ENDED** Write an inequality that you can solve using the Division Property of Inequality where the direction of the inequality symbol must be reversed.

Practice and Problem Solving

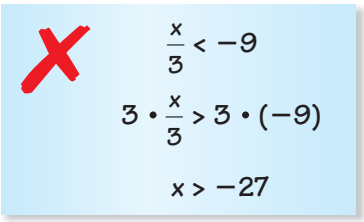
Use a table to solve the inequality.

4. $2x < 2$
5. $-3x \leq 3$
6. $-6x > 18$
7. $\frac{x}{-5} \geq 7$
8. $\frac{x}{-1} > \frac{2}{5}$
9. $\frac{x}{3} \leq \frac{1}{2}$

Solve the inequality. Graph the solution.

10. $2n > 20$
11. $\frac{c}{9} \leq -4$
12. $2.2m < 11$
13. $-16 > x \div 2$
14. $\frac{1}{6}w \geq 2.5$
15. $7 < 3.5k$
16. $3x \leq -\frac{5}{4}$
17. $4.2y \leq -12.6$
18. $11.3 > \frac{b}{4.3}$

19. **ERROR ANALYSIS** Describe and correct the error in solving the inequality.



Write the word sentence as an inequality. Then solve the inequality.

20. The quotient of a number and 4 is at most 5.
21. A number divided by 7 is less than -3.
22. Six times a number is at least -24.
23. The product of -2 and a number is greater than 30.
24. **SMART PHONE** You earn \$9.20 per hour at your summer job. Write and solve an inequality that represents the number of hours you need to work in order to buy a smart phone that costs \$299.

25. **AVOCADOS** You have $9.60 to buy avocados for a guacamole recipe. Avocados cost $2.40 each.

a. Write and solve an inequality that represents the number of avocados you can buy.

b. Are there infinitely many solutions in this context? Explain.

26. **SCIENCE PROJECT** Students in a science class are divided into 6 equal groups with at least 4 students in each group for a project. Write and solve an inequality that represents the number of students in the class.

Solve the inequality. Graph the solution.

3 4 **27.** $-5n \leq 15$

28. $-7w > 49$

29. $-\frac{1}{3}h \geq 8$

30. $-9 < -\frac{1}{5}x$

31. $-3y < -14$

32. $-2d \geq 26$

33. $4.5 > -\frac{m}{6}$

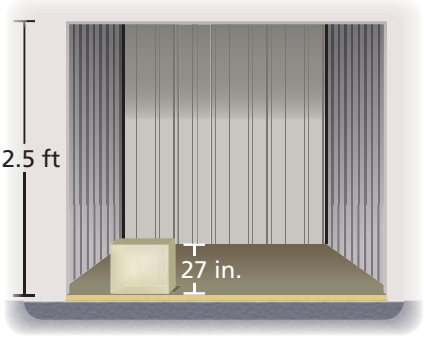
34. $\frac{k}{-0.25} \leq 36$

35. $-2.4 > \frac{b}{-2.5}$

36. **ERROR ANALYSIS** Describe and correct the error in solving the inequality.

$$-3m \geq 9$$
$$\frac{-3m}{-3} \geq \frac{9}{-3}$$
$$m \geq -3$$

37. **TEMPERATURE** It is currently 0°C outside. The temperature is dropping 2.5°C every hour. Write and solve an inequality that represents the number of hours that must pass for the temperature to drop below −20°C.



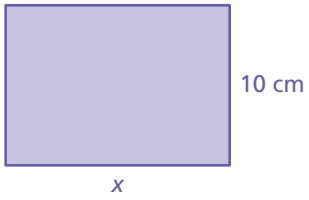
38. **STORAGE** You are moving some of your belongings into a storage facility.

a. Write and solve an inequality that represents the number of boxes that you can stack vertically in the storage unit.

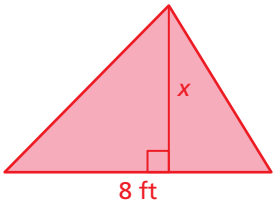
b. Can you stack 6 boxes vertically in the storage unit? Explain.

Write and solve an inequality that represents *x*.

39. Area ≥ 120 cm^2



40. Area < 20 ft^2



41. **AMUSEMENT PARK** You and four friends are planning a visit to an amusement park. You want to keep the cost below $100 per person. Write and solve an inequality that represents the total cost of visiting the amusement park.

42. **LOGIC** When you multiply or divide each side of an inequality by the same negative number, you must reverse the direction of the inequality symbol. Explain why.

43. **PROJECT** Choose two novels to research.

a. Use the Internet or a magazine to complete the table.

b. Use the table to find and compare the average number of copies sold per month for each novel. Which novel do you consider to be the most successful? Explain.

c. Assume each novel continues to sell at the average rate. Write and solve an inequality that represents the number of months it will take for the total number of copies sold to exceed twice the current number sold.

Author	Name of Novel	Release Date	Current Number of Copies Sold
1.			
2.			

Number Sense **Describe all numbers that satisfy *both* inequalities. Include a graph with your description.**

44. $4m > -4$ and $3m < 15$

45. $\frac{n}{3} \geq -4$ and $\frac{n}{-5} \geq 1$

46. $2x \geq -6$ and $2x \geq 6$

47. $-\frac{1}{2}s > -7$ and $\frac{1}{3}s < 12$

Fair Game Review What you learned in previous grades & lessons

Solve the equation. Check your solution. *(Section 3.5)*

48. $-2w + 4 = -12$

49. $\frac{v}{5} - 6 = 3$

50. $3(x - 1) = 18$

51. $\frac{m + 200}{4} = 51$

52. **MULTIPLE CHOICE** What is the value of $\frac{2}{3} + \left(-\frac{5}{7}\right)$? *(Section 2.2)*

Ⓐ $-\frac{3}{4}$ Ⓑ $-\frac{1}{21}$ Ⓒ $\frac{7}{10}$ Ⓓ $1\frac{8}{21}$